Math 131A-3: Homework 1

Due: October 4, 2013

1. Do problems 1.4, 1.8, 1.11, 2.3, 3.7, and 3.8 in Ross.

2. Prove that, for $n \geq 1$, we have the equality $(1 \times 2) + (2 \times 3) + (3 \times 4) + \cdots + [n \times (n+1)] = \frac{n(n+1)(n+2)}{3}$.

3. Prove Bernoulli's inequality: For $x \in \mathbb{R}$ with $1 + x > 0$, and every $n \geq 1$, $(1+x)^n \geq 1 + nx$.

4. Incorrect Inductions

   (a) Consider the following inductive "proof" that all horses are the same color. We will show that any set of n horses have the same color. The base case is trivial, since any set consisting of a single horse has only one color. Now suppose that all sets of $n-1$ horses have only one color. Then if $A = \{x_1, \cdots x_n\}$ is a set of n horses, consider the subsets $A_1 = \{x_1, \cdots, x_{n-1}\}$ and $A_2 = \{x_2, \cdots, x_n\}$. Since each of A_1 and A_2 contain $n-1$ horses, all horses in A_1 must be the same color and all horses in A_2 must be the same color. And these sets overlap, so in fact all horses in A must be the same color. Therefore there is no horse of a different color!

   Explain why this is not a valid inductive proof.

   (b) Consider the following inductive "proof" that all natural numbers are interesting. To begin with, the first case $n = 1$ is clearly satisfied, since 1 is a very interesting number. Next, suppose there are uninteresting natural numbers. Then there must be a smallest such number, call it n. But n is the smallest uninteresting natural number, which is clearly an interesting thing to be! Therefore there aren't any uninteresting natural numbers.

   Explain, in words, why this isn't a valid mathematical proof.